**Specialist Unit 3&4 Test 6 2016**

**SOLUTIONS Calculator Section**

**Time limit = 40 minutes.**

**Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Score: \_\_\_\_\_\_\_\_\_\_ / 34**

**Calculators allowed**

**Access to approved Sample Mathematics Specialist formulae sheet is permitted.**

**Answers should be in 4 decimal places.**

**Q1. [3,3,2]** [8 marks]

The standard deviation of the durability of Performance Racing tyres is 410 kilometres. Racing experts plan to estimate, the mean lifetime of these tyres, using the mean lifetime of a random sample of the tyres.

1. The experts would like to be 95% confident that the mean lifetime of tyres in the sample is within 50 kilometres of . How large a sample should they take?

50 = 1.96 

✓

n = 258.309184

✓

They should take a sample of at least 259 tyres. √

1. Suppose that a random sample of 80 tyres is taken, and the mean lifetime of these tyres is 1245 kilometres. Based on this sample, determine a 90% confidence interval for .

The 90% interval is 1245  1.645 

= (1169.6, 1320.4)

✓

|  |  |
| --- | --- |
| C-Level | 0.90 |
|  | 410 |
|  | 1245 |
| n | 80 |

✓

✓

|  |  |
| --- | --- |
| Lower | 0.90 |
| Upper | 410 |
|  | 1245 |
| n | 80 |

1. The manufacturer claims that the mean lifetime of Performance Racing tyres is at least 1250 kilometres. Does the sample in part (b) provide a strong reason to doubt this claim? Justify your answer.

No, 1250 lies within the 90% confidence interval. We should generally doubt the claim only if 1250 lay outside the confidence interval – and a 90% interval is a narrower interval than others that could be used (say 95%, 99%) to test the claim.1250 would lie well within these intervals.

✓

✓

**Q2. [2,2 ]**  [4 marks]

A machine is designed to produce ball bearings. The quality control officer sampled 400 items and calculated a mean radius of 16 mm and standard deviation of 0.2 mm.

1. Calculate the 99% confidence interval for the population mean of the radii of the ball bearings,

16 – 2.576 x    16 + 2.576 x 

✓

✓

15.974   16.026

1. The company was asked to make ball bearings with radii accurate to within 0.05mm with a 95% confidence. What size sample must be taken to satisfy this request?

 

✓

 62

✓

**Q3. [2,2,1]**  [5 marks]

A survey was conducted via the internet. It asked participants who logged onto a football website; “what fine should a football player receive for major indiscretion?” The statistics of the population resulted in a mean of $4212 and a standard deviation of $1004.

Using a sample of 1256 respondents, the website supervisor calculated the 95% confidence interval to be $4156.47 to $4267.53.

1. Is the confidence level correct? Explain.

4212 - = 4267.53

 Yes it is

4212 + = 4156.47

✓

✓

1. Does this sample represent the view of the population? Explain.

No, only those that use that website

✓

✓

(Or Yes, mean is within interval)

1. Describe the effect on the length of confidence interval if the confidence level were to be increased to 99%

✓

The confidence interval gets wider

**Q4. [1,3,3]**  [7 marks]

If the numbers 1 to 100 form the population, then the mean and standard deviation of the population are 50.5 and 28.87 respectively.

1. Show clearly why the mean of the population is 50.5.



✓

1. If all possible samples of size 9 were taken from this population, what does the Central Limit theorem tell us about the distribution of the means of these samples?

X N 

✓

Or 1. Normally distributed

✓

2. Mean = population mean

3. Standard deviation = 

✓

1. The diagram below shows 3 lists of 9 randomly generated counting numbers from 1 to 100 generated by the calculator

Edit Calc SetGraph

.

|  |  |  |  |
| --- | --- | --- | --- |
|  | List 1 | List 2 | List 3 |
| 1 | 49 | 15 | 75 |
| 2 | 3 | 20 | 70 |
| 3 | 89 | 45 | 72 |
| 4 | 76 | 7 | 19 |
| 5 | 72 | 16 | 74 |
| 6 | 45 | 6 | 81 |
| 7 | 36 | 48 | 83 |
| 8 | 43 | 46 | 27 |
| 9 | 86 | 8 | 11 |
| 10 |  |  |  |
| 11 |  |  |  |
| Calc= randList (9,1,100) | | | |
| Deg Auto Decimal | | | |

Given that the 95% confidence interval for the population mean is 31.64 69.36

Indicate whether these lists above are representative samples, giving your reasons.

✓

 = 55.4 Yes

= 23.4 No

✓

✓

= 56.9 Yes

**Q5. [2,2]** [4 marks]

1. A company wishes to estimate the average age of its employees. From past information it was found the standard deviation was 3.5 years. A sample of 60 employees is selected and the mean is calculated as 32.5 years. Find the 95% confidence interval of the population mean.

The z score for the 95% confidence interval is 1.96

 - z     + z 

✓

32.5 – 1.96 x    32.5 + 1.96 x 

✓

31.6144   33.3856

ie. With a 95% confidence, the average age of employees lies between 26.607 and 28.113 years.

1. How large a sample should the company use to be 95% sure that the sample is within 1 year of the sample mean of 32.5 years,



  47.0596

✓

The required sample size is 48 employees.

✓

**Q6. [1,2,2,2]**  [7 marks]

On the basis of the results obtained from a random sample of 60 bags produced by a mill, the 95% confidence interval for the mean weight of flour in a bag is found to be (3.1kg, 4.6kg).

1. Find the value , the standard deviation of the normal population from which the sample is drawn.

- 1.96     + 1.96 

- 1.96  = 3.1

+ 1.96  = 4.6

✓

solve it simultaneously 0 + 2 = 1.5  = 2.964

1. Find the value of , the mean weight of the sample.

✓

- 1.96  = 3.1

✓

 = 3.850

1. Calculate the 99% confidence interval for the mean weight of flour in a bag.

k = 2.576 for 99% confidence interval

2.864  4.836

✓

✓

1. Using the sample mean from (a) as the best estimate for the population mean, what is the probability that the sample mean of a larger sample of 200 bags is less than 3.8kg.

✓

✓